New approach of diffraction of electromagnetic waves by a rough surface

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We consider the diffraction of a plane wave by a rough surface. In the sake of simplicity the study is restricted to the case of perfectly conducting surfaces. Solving this problem is only possible by limiting the infinite rough surface to a window of width D. We show that we can obtain the diffraction pattern at infinity in the Fraunhofer zone from the modeling of diffraction by a grating with period D whose elementary pattern coincides with the rough surface in the window D. We give some numerical results for triangular profiles or rectified cosine. We show that for small heights we find that the most widely Kirchhoff approximation is very well checked. This modeling can be applied to Fraunhofer diffraction problem by a non-planar metal strip and the complementary problem of diffraction by a perfectly conducting screen, infinitely thin and with a slit of one or more periods.

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I. INTRODUCTION

The problem of diffraction of electromagnetic waves by rough surfaces occurs in many areas: optics, radar detection, telecommunications and more recently in computer graphics for the characterization of textures of materials. If diffractive surfaces are perfectly conducting, the equations of the problem are dimensionless and therefore apply regardless of the frequency used.

According to the geometric nature of the surface methods used to solve are of different types depending on the value of the radius of curvature with respect to the wavelength. They appeal to geometrical optics for very large radii of curvature, physical optics or geometrical theory of diffraction for large curvatures and electromagnetism to the curvatures of the order of magnitude of the length wave. Scalar Kirchhoff approximation theory (KA) is the most widely used theory in the wave scattering from rough surfaces [1,2,3]. The reason is that it is easily understandable physical basis and leads to relatively simple analytical expressions for scattered field amplitudes (in some important limits). The basic feature of the theory is the assumption that the wave field on the surface of a scatterer is approximated as follows: each point of the surface is considered as a point of an infinite plane surface parallel to the local tangent surface.The interaction of the wave field with the surface is treated as the interaction of that field with that plane surface. The KA method gives good results in a rather large domain, but when the roughness becomes large, it appears multiple reflections; then it's necessary to use rigorous electromagnetic methods. In this paper, we propose a new approach to this problem in the resonant domain based on the resolution of the diffraction gratings, surfaces for which there are rigorous numerical models and computer gratings codes (GC) very effective. For the sake of simplicity we have chosen to limit our study to perfectly conducting surfaces.

Our results concern the diffraction of a plane wave by a window of width D of a rough surface. They also apply to the case of diffraction by a metal ribbon of width D and the complementary problem (Babinet's theorem) of the diffraction by an infinitely thin metallic grating with a slit of width D. In the case of surfaces with low roughness height we show that one finds well the results given by KA.

In the first part we recall the basic principles of the study of diffraction by gratings and in the second, those of the aperiodic diffraction surfaces. In the third and in the fourth part we propose a new approach to numerical modeling of the diffraction of a plane wave by a rough surface. In the fifth part we give some numerical results which show that the proposed method, although much more general, is in perfect agreement with (KA).

II. DIFFRACTION BY A GRATING

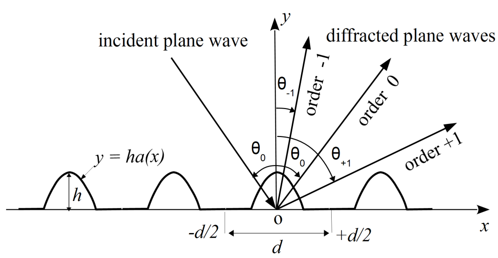

Figure 1. Diffraction by a grating

Diffraction of a plane wave by a grating

Let us consider a grating whose surface coincides with a cylindrical surface, described in rectangular Cartesian coordinate system $Oxyz$ by a periodic profile function $y = ha(x)$ with period d (Fig. 1). The grating is illuminated from the vacuum by an incident monochromatic plane wave with wavelength λ under the incident angle θ_0. The time dependance is $\partial/\partial t \rightarrow i\omega$ where ω is the angular frequency [2,4,5]. Although the method is applicable to any grating, for the sake of simplicity, we consider only the case of perfectly conducting grating.

For 2D problems ($\partial/\partial z \to 0$) there are two kinds of solutions **TE** and **TM.** In both cases the electromagnetic field is completely determined by the knowledge of its component along Oz; we note that component F with $F = E_z$ for **TE** mode and $F = H_z$ for **TM** mode**.** In the classic problem the grating is illuminated by one normalized plane wave, under incidence θ_0: $F^-(x,y) = e^{-ik\alpha_0 x}e^{-ik\beta_0 y}$. Above the grating, $y > \max(ha(x))$, the diffracted wave has the form of a Rayleigh development which consists of a sum of plane and evanescent waves

$$F^+(x,y) = \sum_{n=-\infty}^{+\infty} A_n^+ e^{-ik\alpha_n x}e^{-ik\beta_n y} \quad \text{with } \alpha_n = \alpha_0 + n\frac{\lambda}{d} . \tag{1}$$

If $|\alpha_n| \leq 1$ then A_n^+ correspond to outgoing propagating plane wave with angle θ_n so that $\alpha_n = \sin\theta_n + n\lambda/d$ with $n \in U$:

$$U = \left\{ n \in \mathbb{Z} \middle| \left| \sin\theta_0 + n\frac{\lambda}{d} \right| < 1 \right\}. \tag{2}$$

If $|\alpha_n| > 1$ then A_n^+ correspond to an evanescent wave. With our notations the propagation constant β_n according to Oy is given by:

$$\beta_n = \begin{cases} \sqrt{1-\alpha_n^2} = \cos\theta_n & \text{if } |\alpha_n| \leq 1 \\ -i\sqrt{\alpha_n^2 - 1} & \text{if } |\alpha_n| > 1 \end{cases}. \tag{3}$$

Far from the grating, in the Fresnel zone, evanescent waves vanished, the diffracted field is only composed of a finite set of plane waves with angles θ_n

$$F^+(x,y) = \sum_{n \in U} A_n^+ e^{-ik\sin\theta_n x}e^{-ik\cos\theta_n y}. \tag{4}$$

The boundary conditions on the grating surface are used to calculate the diffracted amplitudes A_n^+ in the directions θ_n and therefore efficiencies in these directions: $\mathscr{E}_n = |A_n^+|^2 \cos\theta_n / \cos\theta_0$.

For numerical modeling we use the C-method or Chandezon method [6,7,8,9] developed in our Laboratory from the late 70s. This is a differential method of solving Maxwell's equations in a curvilinear translation coordinates system (u, v) adapted to the geometry of the surface such that the surface $v = 0$ coincides with the grating surface.

The extended grating problem

In fact, numerical modeling gives the solution of a more general problem called here extended grating problem (EGP), where the incident wave is not a single plane wave, but a discrete sum of plane propagating waves or evanescent waves according α_n correlated with each other, with complex amplitudes A_n^-, whose α_n propagation constants obey to grating equation (Eq. (2)) . For $y > \max(ha(x))$ the total field is written

$$\begin{aligned} F(x,y) &= \sum_{n=-\infty}^{+\infty} A_n^- e^{-ik\alpha_n x}e^{+ik\beta_n y} \\ &+ \sum_{n=-\infty}^{+\infty} A_n^+ e^{-ik\alpha_n x}e^{-ik\beta_n y}. \end{aligned} \tag{5}$$

The first sum represents the incoming waves and the second outgoing waves. In the numerical approximation of order M with $-M \leq n \leq +M$ we keep only $2M+1$ terms in this development. The writing of boundary conditions on the surface $y = ha(x)$ allows to determine the linear relationship between the $2M+1$ amplitudes of the incoming waves A_n^- assumed to be known, and those of the $2M+1$ amplitudes of outgoing waves A_m^+

$$A_n^+ = \sum_{m=-M}^{+M} S_{m,n} A_m^-. \tag{6}$$

The $\mathbf{S}$ matrix with elements $S_{m,n}$ is the diffraction matrix associated to the profile for incident direction θ_0.

If we are only interested by the far field this matrix is restricted to real orders, it is denoted $\mathbf{S}^\infty$ with elements $S_{m,n}$ where $m, n \in U$. We can then write

$$A_n^+ = \sum_{m \in \mathcal{U}} S_{m,n} A_m^-, \quad \text{with: } m, n \in U. \tag{7}$$

the knowledge of $\mathbf{S}^\infty$ is sufficient to determine the far field diffracted by the grating if we know the incident far field. It is not possible to calculate directly $\mathbf{S}^\infty$ but only $\mathbf{S}$. After normalization, efficiencies or energy distribution between the different directions are given by

$$\mathscr{E}_{n,m} = A_{n,m}^+ \overline{A_{n,m}^+} \tag{8}$$

Quantity $\mathscr{E}_{n,m}$corresponds to the diffracted energy in the direction θ_n when the grating is illuminated by a plane wave in the direction θ_m.

Diffraction of a beam by a grating

In the most general problem only a finite portion of the grating is illuminated by the incident wave which is then a beam consisting of a continuous sum of plane waves

$$F^-(x,y) = \int_{\alpha=-1}^{+1} A^-(\alpha) e^{-ik\alpha x}e^{+ik\beta y} \mathrm{d}\alpha. \tag{9}$$

A plane wave with incidence θ_0 is a particular case of beam for which $A^-(\alpha) = \delta(\sin\theta_0 - \alpha)$ where $\delta(x)$ is a Dirac function.

The diffracted wave is also a beam including propagating and evanescent waves

$$F^+(x,y) = \int_{\alpha=-\infty}^{+\infty} A^+(\alpha) e^{-ik\alpha x}e^{-ik\beta y} \mathrm{d}\alpha. \tag{10}$$

Solving the problem is to determine$A^+(\alpha)$ from$A^-(\alpha)$ assumed known. This can be done by discretizing α from an origin α_0 with a regular spacing $\Delta\alpha$ where $\Delta\alpha = \Delta(\sin\theta)$ is the chosen angular resolution. Calculations are then made with the GC for the EGP for all associated $\alpha_0 + n\Delta\alpha$ incidences. This rigorous resolution takes into account diffraction of all the grating patterns. In the following we propose a method of resolution to consider only a single pattern of the grating.

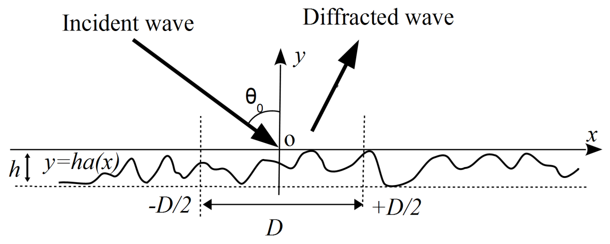

Figure 2. Diffraction by a rough surface

III. Diffraction of a plane wave by an aperiodic surface

Problem formulation

We consider an aperiodic infinite rough surface Σ (Fig. 2), perfectly conducting, with profile $y = ha(x) - y_0$ on which apply the boundary conditions between the incident and the diffracted field. The dimensionless function $a(x)$ is normalized such that $\max(a(x)) - \min(a(x)) = 1$. The parameter h is the maximum amplitude of the modulation.

The plane surface origin $y = 0$ is chosen so as to be immediately above the rough surface then $y_0 = h\max(a(x))$.

We are looking for the diffracted field $F^+(x, y)$ by Σ illuminated by the incident wave $F^-(x, y)$. In Cartesian coordinates the most general solution of Maxwell's equations in the half space $y > 0$ above Σ is a beam which is written in the integral form

$$F^{\pm}(x, y) = \int_{\alpha=-\infty}^{+\infty} A^{\pm}(\alpha) e^{-ik\alpha x} e^{\mp ik\beta y} \mathrm{d}\alpha, \qquad (11)$$

with $\alpha^2 + \beta^2 = 1$ and where $F^{\pm}(x, y)$ is the component of the electromagnetic field along Oz. The $\pm$ sign corresponds respectively to outgoing or incoming waves. If $|\alpha| \leq 1$ then $A^{\pm}(\alpha)$ corresponds to an outgoing or incoming plane propagating wave according to the θ angle with $\alpha = \sin\theta$ and if $|\alpha| > 1$ to an evanescent or anti-evanescent wave whose propagation constant β along Oy is given by Eq. (2) . At infinity it remains only the propagating waves for which $|\alpha| < 1$.

In the surface $y = 0$, the field is

$$F^{\pm}(x) = \int_{\alpha=-\infty}^{+\infty} A^{\pm}(\alpha) e^{-ik\alpha x} \mathrm{d}\alpha. \qquad (12)$$

Amplitudes $A^{\pm}(\alpha)$ are Fourier transform of the field for $y = 0$

$$A^{\pm}(\alpha) = \frac{1}{\lambda} \int_{x=-\infty}^{+\infty} F^{\pm}(x) e^{+ik\alpha x} dx. \qquad (13)$$

The energy radiated by each of the plane waves with angle θ which compose the diffracted beam can be written

$$\mathscr{E}(\theta) = A^+(\theta)\overline{A^+}(\theta). \qquad (14)$$

Solving the problem of diffraction by the rough surface is, writing the boundary conditions, to determine $A^+(\alpha)$ knowing $A^-(\alpha)$.

Classical methods

Most of the time, the problem of diffraction by an aperiodic surface is modeled by the methods of physical optics like KA [1,2] which is to assume that the surface can, at any point, be represented by its plane tangent surface and calculating the wave diffracted by the plan from the Fresnel coefficients. KA gives very good results for the roughness of low amplitude and low slope ie when there are no multiple reflections.

In the case where the dimension of the roughness is of the order of the wavelength, and where there are multiple reflections, it is necessary to use a much more expensive electromagnetic theory in calculation time. In this domain we find many works [10-14].

Maystre et al [15] showed that in their opinion, there are three main methods to solve this problem:

1) to consider that an aperiodic surface is the limit, when the period tends to infinity, of a periodic surface leading to periodize the surface and then solve the problem like a diffraction grating. The grating program gives only a finite and discrete set of diffraction directions angularly separated by $\Delta\sin\theta = \lambda/D$. Therefore, we are led to select a large period to limit the interpolation problems between calculation points.
2) to illuminate by a plane wave over a window of width L of the surface and then solve the problem with a GC considering the incident truncated plane wave as a beam.
3) to illuminate the window L with a Gaussian beam for example, for having an illumination which tends to zero at the edges of the window. In fact this method is a generalization of method 2 to arbitrary beams.

To these three cases, we can add the canonical problem of diffraction of a plane wave by a plan locally deformed where, most of the time, the proposed solutions are type 1. There is an additional difficulty which is the response of the deformation is embedded in the specular reflection of the plane mirror. Then it is necessary to subtract the response of plane mirror to the found solution. The numerical experiment shows that this method is rapidly becoming unusable when the height h increases.

Finally, in all cases, the problem is solved by a periodization then, in addition to the desired solution, this method introduces the influence of multiple diffractions between all patterns of the grating associated with Σ. The influence of these multiple diffractions increases very rapidly with h which limits the field of application of these methods. One way to eliminate the influence of multiple diffractions is to introduce between the patterns an area where we place a fictitious material (PML) eliminating the couplings between the patterns [16].

As for us we propose an interpolation method which easily suprime the multiple diffractions with an optimal modeling of the problem. In the limiting case where the period tends to infinity we find method 1.

IV. New approach to diffraction by an aperiodic surface

Discrete approximation of the electromagnetic field

Except for trivial case $h = 0$, where the surface is a plane mirror there is no analytical solution; then we are led to replace the original continuous problem in α by a discrete problem by discretizing α and then amplitudes $A^{\pm}(\alpha)$ of incident and diffracted fields. By assumption, samples $A_n = A(\alpha_n)$ are taken regularly according to the variable α posing $\alpha_n = \alpha_0 + n\lambda/D$ where D is a parameter that has the dimension of a length.

Around an arbitrary origin according to x, define a spatial window $\widetilde{D}$ (Fig. 2) such as $\widetilde{D} = \{x \in \mathbb{R} \,|\, -D/2 < x < +D/2\}$. Consider the electromagnetic field $F_0(x) = F(x, y = 0)$ in the plan $y = 0$, the discrete representation of the field transforms the aperiodic function in a periodic function of period D

$$F_0(x) = \frac{\lambda}{D} \sum_{n=-\infty}^{+\infty} B_n e^{-ik\alpha_n x}, \; x \in \mathbb{R}. \qquad (15)$$

On the other hand we put

$$F_D(x) = \Pi(x/D) F_0(x), \qquad (16)$$

where $\Pi(x)$ is the rectangular function

$$\Pi(x) = \begin{cases} 1 & |x| \leq 1 \\ 0 & |x| > 1 \end{cases}. \qquad (17)$$

$F_D(x)$ is a function that is zero outside of $x \in \widetilde{D}$ written

$$F_0(x) = F_D(x) + (1 - \Pi(x/D)) F_0(x). \qquad (18)$$

The field $F_D(x)$, can be represented by a Fourier series

$$\begin{cases} F_D(x) = \frac{\lambda}{D} \sum_{n=-\infty}^{+\infty} B_n e^{-ik\alpha_n x} & \text{if } x \in \widetilde{D} \\ F_D(x) = 0 & \text{if } x \notin \widetilde{D}. \end{cases} \qquad (19)$$

Profile representation

We have shown that after discretization the modeled electromagnetic field is periodic with period D; this implies that the geometry that is involved in numerical modeling is also periodic of period D. This leads to build from an aperiodic surface Σ a periodic associated surface Σ' which coincides with Σ in $\widetilde{D}$. It is this surface Σ' with elementary pattern Σ_D for $x \in \widetilde{D}$ which is taken into account in the modeling. Therefore, the numerical modeling is totally identical for all surfaces Σ coinciding in $\widetilde{D}$ leading to the same surface Σ'. This applies in particular to the following three cases:

- diffraction by a locally deformed plan surface defined by $y = f(x) = \Pi(x/D) ha(x)$ (outside $\widetilde{D}$ the surface Σ is a plan),
- diffraction by a metallic ribbon of width D where there is no diffraction outside $\widetilde{D}$,
- diffraction by only one pattern of a periodic surface, ie a grating, so that $y = f(x + D) = f(x)$.

As we have rigorous programs GC for the diffraction of a plane wave by gratings ie for periodic surfaces Σ'. It is from GC that we will deduct the diffraction in the window $\widetilde{D}$ of the rough surface Σ. This model gives the diffracted energy only for λ/D. If the GC is used to calculate additional points for truncated plane wave, additional rigorous results are valid for the infinite grating and do not affect the problem for a single patternΣ_D. If, for example, calculations are made to values such that $\Delta\alpha = \lambda/2D$ the new results concern a $2D$ wide window that takes into account the multiple diffractions between two consecutive patterns Σ_D of Σ' .

Above the associated grating to Σ, for $y > 0$, the electromagnetic field is written in the form of Rayleigh development Eq. (5). For $y = 0$ coefficients A_n^+ of Rayleigh development given by the GC coincide with B_n Eq. (19). If we know all A_n^+ then we know, in the approximation λ/D the diffracted field at any point of $\widetilde{D}$ for $y > 0$.

The different domains for the electromagnetic field

The modeling gives the value of the field in $\widetilde{D}$. As in antenna theory [17] according to the value of y, different domains can be defined for the electromagnetic field:

- the field inside roughness for $ha(x) < y < 0$ which is expressed analytically in terms of curvilinear translation coordinates (u, v) of C-Method,
- the near field for $0 \leq y \lesssim 3\lambda$ which is expressed in Cartesian coordinates system as the sum of plane and evanescent waves,
- the far field for $3\lambda \lesssim y \lesssim D^2/\lambda$ (Fresnel domain) which is expressed as a finite sum of plane waves which interfere with each other,
- the field at infinity for $y \gg D^2/\lambda$ (Fraunhofer domain) where there is no interference at all, in any point the wave is locally a plane wave.

In Fraunhofer domain, at a point P located at the distance r from the origin, in the vicinity of normal incidence the width D of the diffractive element is seen under the angle $\phi \simeq D/r$. If $\phi \ll \lambda/D$ ie $r \gg D^2/\lambda$ then the diffracting element is seen as a point in the retaining approximation. As a result, the diffracted wave by this element is locally plane in the vicinity of P and it propagates in the direction θ, it is then written

$$rF(\overrightarrow{r}) = A(\theta) e^{-i\overrightarrow{k}\cdot\overrightarrow{r}} \text{ si } r \gg D^2/\lambda. \qquad (20)$$

This expression is valid for any point at the distance r from the origin including outside the window$\widetilde{D}$, it is an extrapolation relationship for the field. As it is usual in the antenna theory, for a given r , all the points of the diffracting object being at the same distance r, amplitudes $A(\theta)$ are in phase. On the other hand due to the energy conservation field amplitude decreases as $1/r$.

V. Numerical modeling

In all that follows the incident field consists of a single plane wave with incidence angle θ_0. We choose an angular resolution $\Delta \sin\theta = \lambda/D$, numerical modeling is made from our GC for a grating with period D, whose elementary pattern Σ_D coincides with Σ in $\widetilde{D}$. In Fig. 3 the surface Σ is a plan surface with a triangular deformation. We solve the EGP by writing the boundary conditions on the periodic surface $y = ha(x)$.

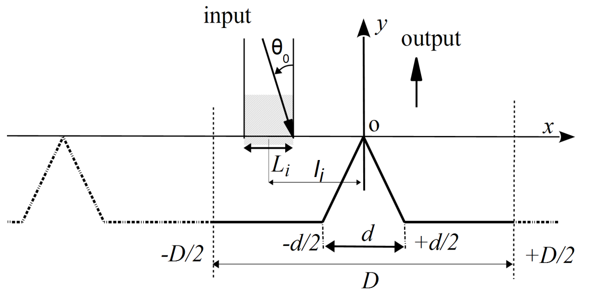

Figure 3. Diffraction of a beam by a grating

Incident field

For $y = 0$ the incident plane wave $F_0^-(x) = e^{-ik\sin\theta_0 x}$ is divided into two parts: the fraction that illuminates a portion of width L_i of $\widetilde{D}$ centered on the abscissa $x = l_i$ and the rest of the plane wave

$$\begin{aligned} F_0^-(x) &= \Pi(\frac{x-l_i}{L_i})F_0^-(x) \\ &+ \left(F_0^-(x) - \Pi(\frac{x-l_i}{L_i})F_0^-(x)\right). \quad (21) \end{aligned}$$

The first term represents a pencil of light of width L_i, the second term represents the complete plane wave that illuminates the entire grating except the window of width L_i (Babinet's theorem [2]). Each of the two terms of Eq. (21) forms a beam. Maxwell's equations are linear, the solution to the diffraction of the plane wave is the sum of solutions obtained for each of the terms. What concerns us here is the solution for the first term, which represents response of $\widetilde{D}$ to excitation by a width of light beam L_i centered at l_i in the plane $y = 0$.

In the Fourier space the first term is:

$$\begin{aligned} F_{L_i}^-(x) &= \Pi(\frac{x-l_i}{L_i})e^{-ik\sin\theta_0 x} \quad (22) \\ &= \int_{\alpha=-\infty}^{+\infty} A^-(\alpha)e^{-ik\alpha x}\mathrm{d}\alpha, \quad (23) \end{aligned}$$

with

$$A^-(\alpha) = \frac{L_i}{\lambda}\mathrm{sinc}(\pi(\alpha-\alpha_0)L_i/\lambda)e^{+i2\pi l_i/D}, \quad (24)$$

where $\mathrm{sinc}(x) = \sin(x)/x$ is the cardinal sine. With the chosen angular resolution using the discretized amplitudes of the incident wave $A_n^- = A^-(\alpha_n)$ are

$$A_n^- = \frac{L_i}{\lambda}\mathrm{sinc}(\pi n L_i/D)e^{+i2\pi l_i/D}. \quad (25)$$

If $L_i = D$ all A_n^- are equal zero except for $n = 0$

Diffracted field by $\widetilde{D}$

For periodic problem associated with $\widetilde{D}$, above the grating ($y > 0$) the diffracted field is written in the form of a Rayleigh development [4]. As we are concerned only by the field to infinity we restrict this development only to plane waves

$$F_\infty^+(x,y) = \sum_{n\in U} A_n^+ e^{-ik\alpha_n x}e^{-ik\beta_n y}. \quad (26)$$

In the plane $y = 0$, each of the plane waves of this development can be considered as the sum of a truncated plane wave $F_{(D),n}^+$of width D and the rest of the plane wave. For each diffraction order, we can write

$$F_{(D),n}^+(x) = \Pi(\frac{x}{D})A_n^+ e^{-ik\alpha_n x}, \quad (27)$$

let, in the Fourier space

$$F_{(D),n}^+(x) = \int_{\alpha=-\infty}^{+\infty} A_n^+(\alpha)e^{-ik\alpha x}\mathrm{d}\alpha, \quad (28)$$

with

$$A_n^+(\alpha) = A_n^+\frac{D}{\lambda}\mathrm{sinc}\left(\pi(\alpha-\alpha_n)D/\lambda\right). \quad (29)$$

A priori modeling gives the electromagnetic field only in the window$\widetilde{D}$ however, to a point at infinity in the Fraunhofer domain at the distance r of origin the diffracted field by $\widetilde{D}$ can be considered as the radiation of a diffracting point located at the origin O; we can then think that we may use the above equation to calculate the field by extrapolation to infinity in the form of a locally plane wave Eq. (20)). In the Fraunhofer domain the amplitude of the plane wave at a point P is the sum of all contributions of each order diffracted by the grating

$$A^+(\alpha) = \sum_{n\in U} A_n^+\frac{D}{\lambda}\mathrm{sinc}\left(\pi(\alpha-\alpha_n)D/\lambda\right), \quad (30)$$

let

$$A^+(\theta) = \sum_{n\in U} A_n^+\frac{D}{\lambda}\mathrm{sinc}\left(\pi(\sin\theta-\sin\theta_n)D/\lambda\right). \quad (31)$$

At the angular precision related to D, in Fraunhofer domain, for a given value of r the energy density radiated in the direction θ is then

$$\mathscr{E}(\theta) = A^+(\theta)\overline{A^+(\theta)}. \quad (32)$$

We note that if $\theta = \theta_n$ then $A^+(\theta_n) = A_n^+$, diffracted energy in such direction θ_n is equal to the efficiency of the associated grating.

VI. Results

Results are given for an aperiodic surface, we consider only the window $\widetilde{D}$. This surface is illuminated by a plane wave of incidence θ_0, we compute the diffracted wave at infinity created by only the part of the incident plane wave that illuminates in a plane $y = 0$ the part of width L_i of $\widetilde{D}$ centered on l_i(Fig.3).

Principle of calculation

In summary, from the GC the resolution of diffraction problem by a part of window $\widetilde{D}$ of the rough surfaceΣ is done in five steps:

1) determination of the diffraction matrix $\mathbf{S}^\infty(\theta_0)$ of periodized problem with elementary pattern Σ_D for the angular resolution $\Delta\sin\theta = \lambda/D$ with incidence θ_0,
2) computation of A_n^- for the incident plane wave truncated to width L_i centered on a point located at the distance l_i of the origin Eq. (24).

3) computation of A_n^+ Eq. (7) of the diffracted wave to infinity by the grating of period D ,
4) calculation of the analytical expression of the diffracted amplitude $A^+(\theta)$ by interpolation in cardinal sine from A_n^+computed in step 3, Eq. (30),
5) calculation of the angular distribution of diffracted energy $\mathscr{E}(\theta)$ normalized with respect to energy of the incident beam where angles are given in degrees.

Diffraction by a deformed plane surface

Deformation of small height in Kirchhoff method domain

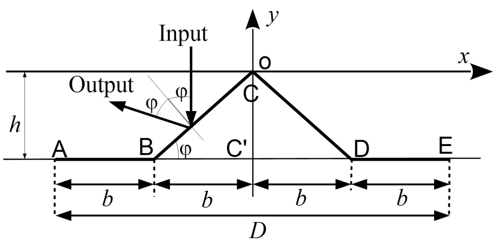

Figure 4. A plan with a triangular deformation

To be able to simply compare our results with those that would be obtained in the Kirchhoff method the chosen deformation ABCDE consists of four line segments Fig. 4. To limit the number of parameters and simplify the interpretation of results we chose AB=BC'=C'D=DE=$b = 3.2\lambda$ and a normal incidence $\theta_0 = 0°$. The geometrical optics shows that up to a slope $\varphi = 45°$ there is no double reflection on the entire pattern, this is the Kirchhoff domain where we have chosen the first example with $h = 1.848\lambda$ and $\varphi = 30°$ Fig. 5.

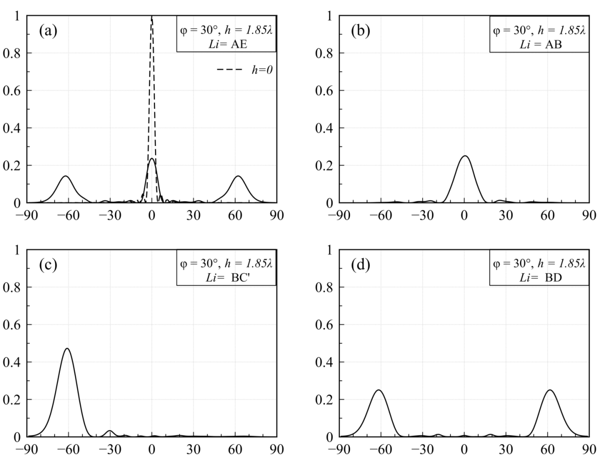

Figure 5. Diffraction for small height h= 1.848λ, (a) diffraction by the complete pattern, with dotted line diffraction for $h = 0$, (b) diffraction by AB, (c) diffraction by BC and (d) diffraction by BD

In this example the two polarizations give practically the same results. We represent the reflection from the plane surface $D =$ AE dotted line corresponding to $h = 0$ and full line corresponding to the diffraction for both **TE** and **TM** for $h = 1.848\lambda$.

- Fig. 5a: the entire pattern is illuminatedby the plane wave ($L_i =$ AE, $l_i = 0$) . We note the presence of three maximum : $\theta = 0$ for the reflections on AB and DE and two maximum near $\theta = \pm 60°$ for reflection on inclined plane surfaces CD and BC.
- Fig. 5b: only side AB is illuminated ($L_i = b$, $l_i = -3/2b$), specular reflection is almost perfect.
- Fig. 5c: only the inclined side BC is illuminated ($L_i = b$, $l_i = -$b/2), there again it almost perfectly specular reflection for incidence of 30 ° which give $\theta = 60°$.
- In Fig. 5d: both sides BC and CD are illuminated ($L_i = 2b$, $l_i = 0$), once again reflections are around $\theta = \pm 60°$

These results show that, as expected, KA is well verified, replacing the profile by four planar segments led to very good results.

Deformation with multiple reflections

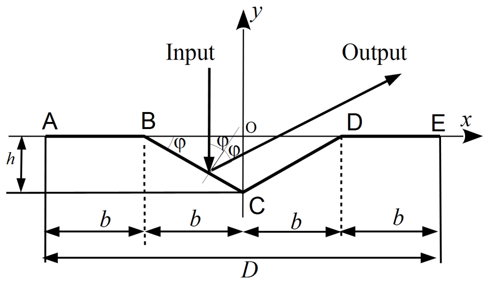

Figure 6. A plan with a triangular groove

We keep the same geometry but with a hollow deformation Fig.6 that will lead to multiple reflections. The geometrical optics shows that the double reflection appears from $\varphi = 30°$. For $\varphi = 45°$ after a double reflection, light comes back into the direction of incidence. This phenomenon occurs again at $\varphi = 60°$ after a triple reflection. The numerical experiment shows that the horizontal faces AB and DE on which there is no double reflection always reflect as a horizontal plane mirror. It is for this reason that we have represented only the reflection on the inclined face CD ($L_i = b$, $l_i = -b/2$) for $\varphi = 30°, 45°, 60°, 75°$.

- Fig. 7a: $\varphi = 30°$, it is observed that the result is practically the same as in Fig. 5c, there is little coupling with the other faces.
- Fig. 7b :$\varphi = 45°$, we observe that, with the double reflection, as expected, most of the light comes back into the direction of incidence $\theta = 0$ which is still the case for $\varphi = 60°$ Fig. 7c.
- Fig. 7d: $\varphi = 75°$ the effects of reflections triple and quadruple completely distort the specular reflection by CD and the effects of polarization have a important effect.

In the last three cases the method Kirchhoff is unable to correctly model the diffraction.

Diffraction by one or more patterns of a grating

The rough surface Fig.8, is a grating with period d with $a(x) = -\left|\cos(2\pi x/d)\right|$. We consider a part $D = Nd$ of

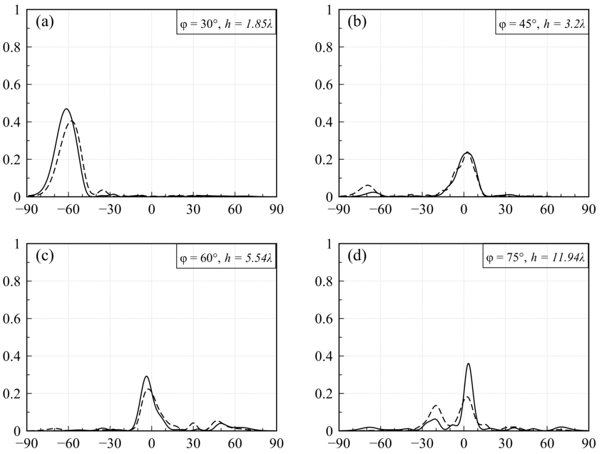

Figure 7. Diffraction of a triangular groove for several values of φ, when only inclined face CD is illuminated, dotted line **TE** mode, solid line **TM** mode.

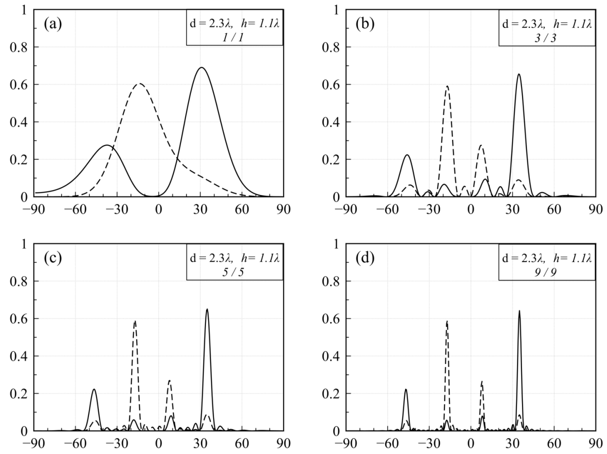

Figure 9. Diffraction of a plane wave by a planar finite surface of width $D = Nd$ with N=1,3,5,9 identical patterns of width d, doted line **TE** mode, solid line **TM** mode

this grating equal to an integer number of periods. The selected operating point is in the resonant domain : $d = 2.3\lambda$, $h = 1.1\lambda$ and $\theta_0 = 8°$. For this grating there are four real diffraction orders (-2,-1,0,+1). Efficiencies for this grating are given in Table 1. We chose this operating point in the resonant domain to have very different results in TE and TM.

orders	angles	efficiencies **TE**	efficiencies **TM**
-2	-46.92°	0.0491	0.2205
-1	- 17.19°	0.5900	0.0535
0	8.00°	0.2716	0.0703
1	35.03°	0.0894	0.6557

Table I
EFFICIENCIES FOR GRATING WITH PROFILE IN RECTIFIED COSINE

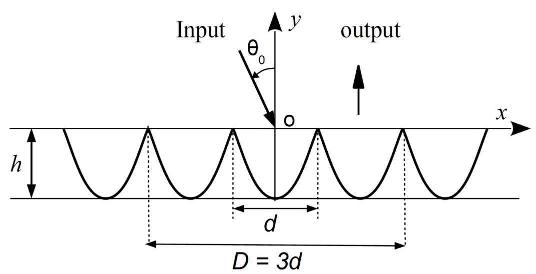

Figure 8. Grating in rectified cosine

Several patterns of the grating are illuminated

We study the diffraction of the fraction of the incident plane wave that illuminates the calculation window of width $D = Nd = L_i$ with $N = 1, 3, 5, 9$ respectively Figs. 9a, 9b, 9c, 9d. We observe the expected result : the wave is diffracted around the grating diffraction orders with the more sharp peaks that N is large.

Only one pattern of the grating is illuminated

We study the diffraction where only the central pattern is illuminated ($L_i = d,\ l_i = 0$) among N patterns regarded in calculation window ($D = Nd$). In Figs. 10a, 10b, 10c, 10d we see the effect of multiple diffractions which lead to the deformation of the initial figure obtained for $N = 1$, Fig. 9a. From $N = 9$, Fig. 10c the diffraction curves do not vary at all, this shows that from this value of N there is no coupling between the central motif and extreme patterns.

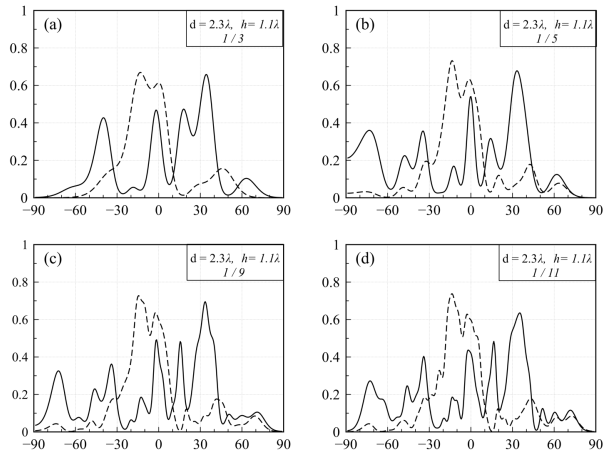

Figure 10. Diffraction of a truncated plane wave by a surface of width $D = Nd$ with $N = 3, 5, 9, 11$ identical patterns of width d where only the central pattern is illuminated, doted line **TE** mode, solid line **TM** mode**.** Results for $N = 1$ are given in Fig. 9a.

Fraunhofer diffraction by a metallic ribbon

We have chosen to determine the diffraction of a portion of width D of a perfectly conductive surface, the results relate only diffraction by this surface portion regardless of what exists outside this window. Hence the Fraunhofer diffraction pattern obtained is the same that for an infinitely thin metal ribbon of width D having the form of a single grating pattern. By Babinet's theorem is also diffraction obtained for the complementary problem with a perfectly conducting periodic screen where we practiced a slot of width of one pattern, the diffraction pattern is then the same as that of the ribbon except for the angles θ_n where we find efficiencies for the infinite grating. In both cases of geometry discussed in this paper results for the ribbon can be found in Figs. 5a and 9a.

CONCLUSION

Currently, there are rigorous numerical modeling to determine the diffraction of a plane wave by a grating. One may think that the results obtained for a grating formed by the infinite repetition of an elementary pattern contain the results of diffraction of the plane wave by only one of those patterns. We adopted this approach to investigate the diffraction of a plane wave by a window of width D of a rough surface considered as single pattern of a periodic surface with period D. Our numerical results show that one finds well the expected results that are given by geometrical optics, the ray tracing and optical physics.

The results of Fig. 5 concerning a weakly modulated plane surface are in perfect agreement with those provided by KA which assumes that surface is plane in the vicinity of each point. When the height of roughness increases, multiple reflections appear and when both sides of the pattern are at right angles we find the phenomena of retro reflection provided by geometrical optics, this phenomenon persists in the resonant domain as can be seen in Fig. 7.

We used the linearity of Maxwell's equations to decompose the incident plane wave and the diffracted plane waves as the sum of a truncated plane wave and its complement. This is in perfect accord with the Huygens Fresnel principle of assuming that at each point the wavefront can be considered as the limit of isotropic radiation point source. If in our modeling we decompose the incident plane wave into a sum of truncated plane waves with very small width L_i and if then searches the diffracted wave as the sum of all these contributions then we found the Huygens Fresnel principle.

A final argument in favor of our approach lies in the Heisenberg uncertainty principle [18]. If we are interested in the corpuscular aspect of light it is possible to interpret the efficiency of a grating such as the probability of finding in the direction θ_n an incident photon associated to a plane wave arriving under the incidence θ_0. If one restricts the study of the diffracted wave only to incident photons which interact with a single period D of the grating they are diffracted in a direction θ with a probability density $A(\theta)\overline{A(\theta)}$. There is then a information on the position of the diffracted photons which come from a point of the pattern $(-D/2 < x < D/2)$ which, by virtue of the uncertainty principle, induces an uncertainty in their direction. In the plan $y = 0$ the uncertainty relation is written $\Delta p_x.\Delta x \geq h$ where h is Planck's constant, Δp_x the uncertainty of the momentum of the photon according Ox and $\Delta x = D$ the uncertainty of the position. For a photon momentum is written $p = h\nu/c = h/\lambda$ with $\Delta p_x = \Delta(p\sin\theta) = (h/\lambda)\Delta\sin\theta$ which leads to an angular uncertainty $\Delta\sin\theta \geq \lambda/D$. From the point of view of the uncertainty principle, once fixed the width D, to make additional measures for an angular resolution $\Delta(\sin\theta)$ less than λ/D adds no additional information. It is therefore possible to determine $A(\theta)$ only from measures $A_n = A(\theta_n)$ made for angles obeying the grating formula $\sin\theta_n = \sin\theta_0 + n\lambda/D$, where θ_0 is an arbitrary origin angle.

If the uncertainty principle should be applied to measurements, it also applies to numerical modeling. If we are looking for the Fraunhofer diffraction pattern of a window of width D illuminated by a plane wave, all information that is possible to know is contained in the diffracted amplitudes in directions θ_n of associated grating. Make additional calculations provides no additional information for diffraction by the window of width D. If, for example, calculations are made to an angular resolution $\Delta\sin\theta = \lambda/2D < \lambda/D$ new results obtained are only valid for $L \geq 2D$, they contain the contributions of multiple reflections between two patterns which is not consistent to the originally posed problem. If the roughness are of very low amplitude effect of multiple reflections is negligible then, in this case, the new calculated points correspond to the problem initially posed however, the new calculations are completely useless.

For sake of simplicity we have chosen a perfectly conducting rough surface, currently, taking in account the conductivity is not a problem in particular in C-Method. The extension of the formalism to the finite conductivity should be done without any difficulty.

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